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Technical Notes

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Correlation Between Dual-Phase-Lag Model and Parabolic Two-Step Model

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Nomenclature

C	=	volumetric heat capacity, $J/(m^3\,K)$
C_e	=	volumetric heat capacity of electron gas, $J/(m^3\,K)$
C_l	=	volumetric heat capacity of metal lattice, $J/(m^3\,K)$
G	=	electron-phonon coupling factor, $W/(m^3\,K)$
K	=	thermal conductivity of the electron gas, $W/(m\,K)$
Q	=	volumetric heating source, W/m^3
Q_e	=	volumetric heating source of the electron gas, W/m^3
Q_l	=	volumetric heating source of the metal lattice, W/m^3
T	=	temperature, K
T_e	=	electron temperature, K
T_l	=	lattice temperature, K
α	=	thermal diffusivity, m^2/s
β	=	coefficient of the heating-source term
τ_T	=	relaxation time of the temperature gradient, s
τ_q	=	relaxation time of heat flux, s

I. Introduction

ULTRASHORT laser-pulse heating of metallic film can result in complicated temperature distributions, because the excited free electrons are not in thermal equilibrium with the surrounding lattice. This nonequilibrium heating phenomenon, with a period of thermalizing time, causes two different temperature distributions in the film. Currently, the parabolic two-step (PTS) model [1] is often used to evaluate the thermal transport phenomenon in metals [2,3]. By introducing two relaxation parameters (namely, the phase lags of temperature gradient and heat flux), Tzou [4] introduced the dual-phase-lag (DPL) model. Tzou [5] evaluated the experimental results of Qiu et al. [3] and Brorson et al. [2] about gold films with the DPL model. Because the PTS and the DPL models can describe the nonequilibrium heating in metals, there may be some correlation between them.

In this Note, we study the correlation between the DPL model and the PTS model. We find that the DPL model is not consistent with the PTS model in describing the nonequilibrium heating in metals. The temperature described by the DPL model is neither the electron temperature nor the lattice temperature.

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II. Analysis

In the analysis, all the thermophysical properties are assumed to be temperature-independent. According to [5], the PTS model without a heating-source term can be expressed as

$$C_e \frac{\partial T_e}{\partial t} = K\nabla^2 T_e - G(T_e - T_l) \tag{1}$$

$$C_l \frac{\partial T_l}{\partial t} = G(T_e - T_l) \tag{2}$$

The DPL model without a heating-source term can be expressed as [5]

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}[\nabla^2 T] = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \tag{3}$$

Combining Eq. (1) with Eq. (2), we can obtain two single energy equations about the electron temperature and the lattice temperature, which are expressed as Eqs. (4) and (5), respectively:

$$\nabla^2 T_e + \frac{C_l}{G}\frac{\partial}{\partial t}[\nabla^2 T_e] = \frac{C_l + C_e}{K}\frac{\partial T_e}{\partial t} + \frac{C_l C_e}{KG}\frac{\partial^2 T_e}{\partial t^2} \tag{4}$$

$$\nabla^2 T_l + \frac{C_l}{G}\frac{\partial}{\partial t}[\nabla^2 T_l] = \frac{C_l + C_e}{K}\frac{\partial T_l}{\partial t} + \frac{C_l C_e}{GK}\frac{\partial^2 T_l}{\partial t^2} \tag{5}$$

From the comparison of Eq. (3) with Eq. (4) or Eq. (5), a perfect correlation results can be obtained using the following substitution [5]:

$$\alpha = \frac{K}{C_l + C_e}, \qquad \tau_T = \frac{C_l}{G}, \qquad \tau_q = \frac{1}{G}\left[\frac{1}{C_l} + \frac{1}{C_e}\right]^{-1} \tag{6}$$

It is shown that for a system without heating source, the PTS model and the DPL model are equivalent. Neither model predicts nonequilibrium thermal transport. And there is actually only one temperature system in this case. In the PTS model, the crucial parameter that induces the difference between the electron and lattice temperature lies in the heating-source term. It then becomes a question of whether the DPL model provides the electron temperature or the lattice temperature. The PTS model with a heating-source term can be expressed as [1]

$$C_e \frac{\partial T_e}{\partial t} = K\nabla^2 T_e - G(T_e - T_l) + Q \tag{7}$$

$$C_l \frac{\partial T_l}{\partial t} = G(T_e - T_l) \tag{8}$$

We find the two equations describing the two kinds of temperature have different heating-source term. From Eqs. (6–8), we can get the individual energy equations about the electron temperature and the lattice temperature as the following:

$$\nabla^2 T_e + \tau_T \frac{\partial}{\partial t}[\nabla^2 T_e] + \frac{1}{K}\left[Q + \tau_T \frac{\partial Q}{\partial t}\right] = \frac{1}{\alpha}\frac{\partial T_e}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T_e}{\partial t^2} \tag{9}$$

$$\nabla^2 T_l + \tau_T \frac{\partial}{\partial t}[\nabla^2 T_l] + \frac{1}{K} Q = \frac{1}{\alpha}\frac{\partial T_l}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T_l}{\partial t^2} \tag{10}$$

Equations (9) and (10) have been derived also by Zhang [6]. We can see that the energy equations of electron gas and lattice no longer have the same form, due to the nonequilibrium between the electron and the lattice temperature. The DPL model with a heating-source term is expressed as

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}[\nabla^2 T] + \frac{1}{K}\left[Q + \tau_q \frac{\partial Q}{\partial t}\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \tag{11}$$

The preceding equation was originally derived by Tzou [5]. Clearly, the differential equation described from the DPL model is identical neither with the electron nor the lattice temperature Eqs. (9) and (10), given by the PTS model.

The basic difference between the two models lies in the heating-source term. In the PTS model, the 'two-step' term means that the radiation energy is only absorbed by the free electrons in the first step, and redistributed to the lattice by the phonon–electron interaction. So, the heating-source term appears only in Equation describing the electron temperature in the PTS model. And it is just the nonequilibrium distribution of the heating source that leads the nonequilibrium heating. However, what is the temperature described by the DPL model? To answer this question, we consider the following distribution of the heating-source term:

$$Q_e = \beta Q, \qquad Q_l = (1-\beta) Q \tag{12}$$

where $0 \le \beta \le 1$. If we introducing Eq. (12) into the PTS model, the energy equations can be rewritten as

$$C_e \frac{\partial T_e}{\partial t} = K\nabla^2 T_e - G(T_e - T_l) + \beta Q \tag{13}$$

$$C_l \frac{\partial T_l}{\partial t} = G(T_e - T_l) + (1-\beta) Q \tag{14}$$

From Eqs. (6), (13), and (14), we can get the single energy equations about the electron and lattice temperature as the following:

$$\nabla^2 T_e + \tau_T \frac{\partial}{\partial t}[\nabla^2 T_e] + \frac{1}{K}\left[Q + \beta \frac{C_l}{G}\frac{\partial Q}{\partial t}\right] = \frac{1}{\alpha}\frac{\partial T_e}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T_e}{\partial t^2} \tag{15}$$

$$\nabla^2 T_l + \tau_T \frac{\partial}{\partial t}[\nabla^2 T_l] + \frac{1}{K}\left[Q + (1-\beta)\frac{C_e}{G}\frac{\partial Q}{\partial t}\right] - \frac{1}{G}(1-\beta)\nabla^2 Q = \frac{1}{\alpha}\frac{\partial T_l}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T_l}{\partial t^2} \tag{16}$$

It can found that only Equation of electron temperature is similar to the DPL model in the case of $\beta \neq 1$. If β equals one, Eqs. (13) and (14) are the PTS model. Comparing Eq. (15) with Eq. (11), we find that only if $\beta\tau_T = \tau_q$, Eq. (15) is equivalent to Eq. (11). Then, we can deduce that

$$\beta = \frac{C_e}{C_e + C_l} \tag{17}$$

From Eq. (17), two heating-source terms in the two temperature systems can be divided as

$$Q_e = \frac{C_e}{C_e + C_l} Q, \qquad Q_l = \frac{C_l}{C_e + C_l} Q \tag{18}$$

When the heating source is distributed by the ratio given in Eq. (18), the DPL model has the same form with the single energy equation described from the PTS model about the electron temperature given in Eq. (15). However, this kind of distribution should not be physical. Therefore, the temperature described by the DPL model is not the actual electron temperature.

III. Conclusions

The DPL model is not consistent with the PTS model in evaluating the thermal transport phenomena in metals heated by ultrashort laser pulses. And the temperature described by the DPL model with a heating-source term is neither the lattice temperature nor the actual electron temperature. The DPL model has some difficulties in evaluating the thermal transport phenomena in metals heated by ultrashort laser pulses.

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